Turn in the following problems:

1. Let $f(x) = x^{-2}$. Your classmate is using the limit definition of the derivative to evaluate $f'(a)$. Your classmate's work is below:

   (i) $\lim\limits_{h\to 0} \dfrac{(a+h)^{-2} - a^{-2}}{h}$

   (ii) $= \lim\limits_{h\to 0} \dfrac{\frac{1}{(a+h)^2} - \frac{1}{a^2}}{h}$

   (iii) $= \lim\limits_{h\to 0} \dfrac{\frac{a^2-(a+h)^2}{a^2(a+h)^2}}{h}$

   (iv) $= \lim\limits_{h\to 0} \dfrac{\frac{a^2-a^2+ah+h^2}{a^2(a+h)^2}}{h}$

   (v) $= \lim\limits_{h\to 0} \dfrac{\frac{ah+h^2}{a^2(a+h)^2}}{h}$

   (vi) $= \lim\limits_{h\to 0} \dfrac{a+h}{a^2(a+h)^2}$

   (vii) $= \dfrac{a}{a^2(a)^2} = a^{-3}$

   If your classmate made any errors in their work, state between which two lines where the error(s) occurred (e.g.,between line (i) and line (ii)), explain the error(s), and then correctly find $f'(a)$.
   If your classmate made no errors in their work, then write 'Excellent work, that was a challenging problem!'.

2. Sketch the graph of an example of a function f that satisfies all of the given conditions:

   - $\lim\limits_{x\to 2} f(x) = \infty$
   - $\lim\limits_{x\to -2^+} f(x) = \infty$
   - $\lim\limits_{x\to -2^-} f(x) = -\infty$
   - $\lim\limits_{x\to -\infty} f(x) = 0$
   - $\lim\limits_{x\to \infty} f(x) = 0$
   - $f(0) = 0$

3. Sketch the graph of a function f for which $f(0) = 0, f'(0) = 3, f'(1) = 0$, and $f'(2) = -1$.

4. Let $h(t)$ be the **change in height** of the tide at the Bay of Fundy in meters since midnight, where t measured in hours. Interpret the following mathematical statements in terms of its physical meaning. Be sure to use units.
(Note: We restrict the domain of $h(t)$ so that h is a one-to-one function.)
   (a) $h(7) = 2.75$
   (b) $h'(7) = 0.21$
   (c) $h^{-1}(-1.5) = 13.2$

5. Let $V(t)$ be the volume of water in a tank (in liters), at time t (in seconds).
   (a) What is the physical meaning $\frac{dV}{dt}$? Be sure to include units.
   (b) The tank is full at time t_0, so that $V(t_0) > 0$. At some later time t_1, a drain is opened 20 cm above the bottom of the tank (which is taller than 20cm), emptying water from the side of the tank. Is $\frac{dV}{dt}$ positive, negative, or zero at the following times?
      i. at any time t where $t_0 < t < t_1$?
      ii. after the drain has been opened at t_1, but before the water has dropped to 20 cm above the bottom of the tank?
      iii. after the water drops to the drain hole 20 cm above the bottom of the tank?

6. The **derivative of a function f at a number** a denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

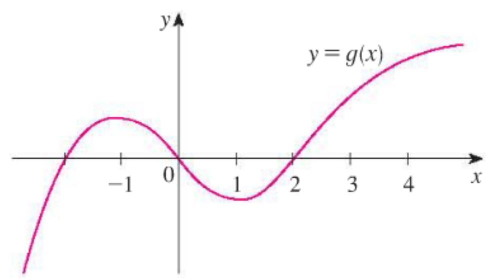
if this limit exists.

Sketch $f(x) = x^2$ and draw a representation that shows the relationship between $f'(a)$, $f(a+h)$ and $f(a)$. Explain how your illustration represents the definition of the derivative a function at a number a.

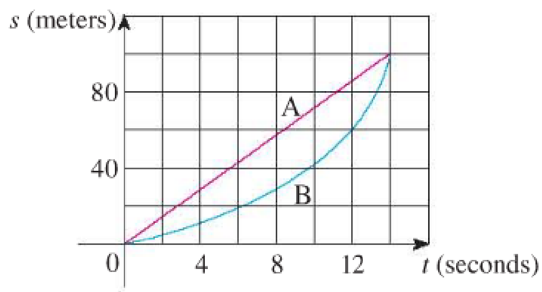
These problems will not be collected, but you might need the solutions during the semester:

1. For the function g whose graph is given, arrange the following numbers in increasing order and explain your reasoning:

    - 0
    - $g'(-2)$
    - $g'(0)$
    - $g'(2)$
    - $g'(4)$



2. Shown are graphs of the position functions of two runners, A and B, who run a 100-m race and finish in a tie.

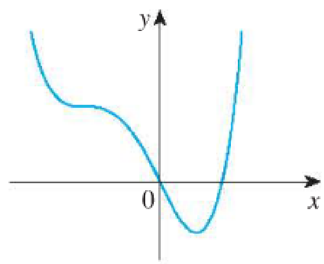


    (a) Describe and compare how the runners run the race.
    (b) At what time is the distance between the runners the greatest?
    (c) At what time do they have the same velocity?

3. Use the limit definition of the derivative to find $f'(1)$ given the following function:

$$f(x) = \frac{1}{1+x}$$

4. Trace or copy the graph of the given function f. (Assume that the axes have equal scales.) Then use the method outlined in Example 1 on page 146 of your text to sketch the graph of f' below it.



5. Let $P(t)$ be the percentage of Americans under the age of 18 at time t. The table gives values of this function in census years from 1950 to 2000.

t	$P(t)$	t	$P(t)$
1950	31.1	1980	28.0
1960	35.7	1990	25.7
1970	34.0	2000	25.7

(a) What is the meaning of $P'(t)$? What are its units?

(b) Construct a table of estimated values for $P'(t)$.

(c) Graph P and P'.

(d) How would it be possible to get more accurate values for $P'(t)$?